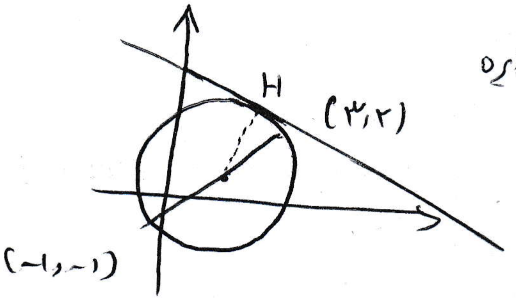
۱-



O مرکز دایره $\boxed{(1, \frac{1}{2})}$

$2R = \sqrt{16+9} = 5 \Rightarrow \boxed{R = \frac{5}{2}}$

$OH = \frac{|3+2-a|}{\sqrt{9+16}} = \frac{5}{2} \Rightarrow 5-a = \pm\frac{25}{2}$

$\to 5-a=\frac{25}{2} \to \boxed{\frac{-15}{2}=a}$

$\to 5-a=-\frac{25}{2} \to \boxed{\frac{35}{2}=a}$

۲-

$mx^2-(m+3)x+5=0$

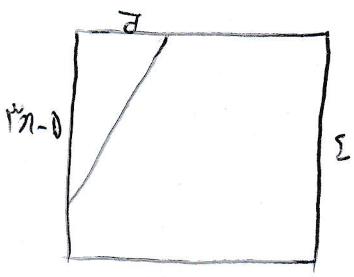
$x_1^2+x_2^2=6 \Rightarrow S^2-2P=6 \Rightarrow \left(\frac{m+3}{m}\right)^2-\frac{10}{m}=6 \Rightarrow$

$m^2+9+6m-10m=6m^2 \Rightarrow 5m^2+4m-9=0 \Rightarrow \begin{cases} m=1 \ \times \\ \boxed{m=-\frac{9}{5}} \end{cases}$

اگر $m=1 \to x^2-4x+5=0 \ \times$

اگر $m=-\frac{9}{5} \Rightarrow -\frac{9}{5}x^2-\frac{6}{5}x+5=0 \to 9x^2+6x-25=0 \ \checkmark \ \Delta>0$

۳-



$3x-5+y=4 \to \boxed{y=-3x+9}$

$S_\Delta = \frac{y(3x-5)}{2} = \frac{(-3x+9)(3x-5)}{2} \Rightarrow$

$S=-\frac{9}{2}x^2+21x-\frac{45}{2} \to -\frac{b}{2a}=\frac{-21}{-9}=\boxed{\frac{7}{3}}$

$x=\frac{7}{3} \to S_{Max} = -\frac{9}{2}\left(\frac{49}{9}\right)+21\left(\frac{7}{3}\right)-\frac{45}{2} = -\frac{49}{2}+49-\frac{45}{2}=\boxed{2}$

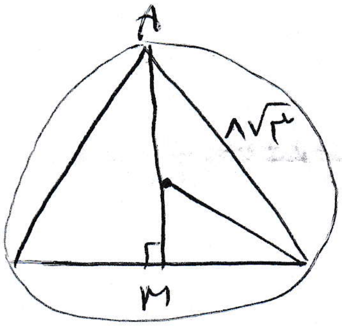
۴-

$\sqrt{x^2+4x+5}=(x+1)(x+3) \to \sqrt{x^2+4x+5}=x^2+4x+3 \to$

$x^2+4x+3=A$ $\quad \sqrt{A+2}=A \xrightarrow{\text{توان ۲}} A+2=A^2 \to A^2-A-2=0 \quad \begin{cases} A=-1 \\ A=2 \end{cases}$

$\begin{cases} x^2+4x+3=-1 \to x^2+4x+4=0 \to \boxed{x=-2} \ \times \\ x^2+4x+3=2 \to x^2+4x+1=0 \to \boxed{x=-2\pm\sqrt{3}} \ \checkmark \end{cases}$

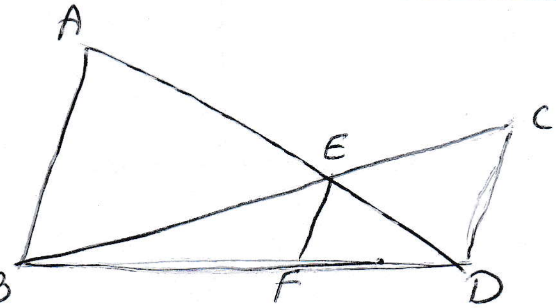
۵ـ در مثلث متساوی الاضلاع، میانه و عمود منصف برهم منطبق هستند



$AM = \frac{\sqrt{3}}{2}(8\sqrt{3}) = \boxed{12}$ طول میانه

شعاع دایره محیطی $= \frac{2}{3} \times 12 = \boxed{8}$

۶ـ



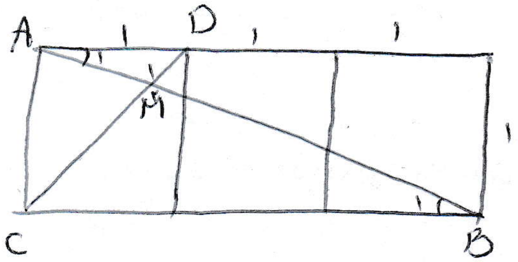
$\overset{\triangle}{ABD}: \frac{EF}{AB} = \frac{FD}{BD} \quad (EF \parallel AB)$

$\overset{\triangle}{BCD}: \frac{EF}{CD} = \frac{BF}{BD} \quad (EF \parallel CD)$

جمع دو طرف $\longrightarrow \frac{EF}{AB} + \frac{EF}{CD} = \overbrace{\frac{FD}{BD} + \frac{BF}{BD}}^{1} \Longrightarrow EF\left(\frac{1}{AB} + \frac{1}{CD}\right) = 1 \Longrightarrow$

$\boxed{\frac{1}{AB} + \frac{1}{CD} = \frac{1}{EF}}$

۷ـ



$AB^2 = 9 + 1 = 10 \Longrightarrow \boxed{AB = \pm\sqrt{10}}$

$\hat{A}_1 = \hat{B}_1$ (موازی مورب)

$\hat{M}_1 = \hat{M}_2$ (متقابل به رأس)

$\Longrightarrow \overset{\triangle}{AMD} \sim \overset{\triangle}{MCB}$

$\longrightarrow \frac{AD}{BC} = \frac{AM}{MB} \Longrightarrow \frac{1}{3} = \frac{AM}{\sqrt{10} - AM} \Longrightarrow \sqrt{10} - AM = 3AM$

$\longrightarrow 4AM = \sqrt{10} \longrightarrow \boxed{AM = \frac{\sqrt{10}}{4}}$

۸ـ

$y = \frac{|x|}{2} - \left[\frac{x}{2}\right] \quad -2 < x < 3 \Longrightarrow -1 < \frac{x}{2} < \frac{3}{2}$

$-1 < \frac{x}{2} < 0 \longrightarrow -2 < x < 0 \longrightarrow y = -\frac{x}{2} + 1$

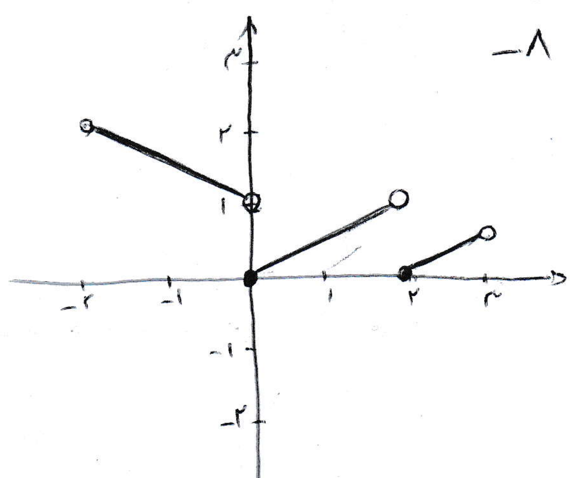
x	-2	0
y	2	1

$0 \le \frac{x}{2} < 1 \longrightarrow 0 \le x < 2 \longrightarrow y = \frac{x}{2}$

x	0	2
y	0	1

$1 \le \frac{x}{2} < \frac{3}{2} \longrightarrow 2 \le x < 3 \longrightarrow y = \frac{x}{2} - 1$

x	2	3
y	0	$\frac{1}{2}$



$R = [0, 2) - \{1\}$

9-

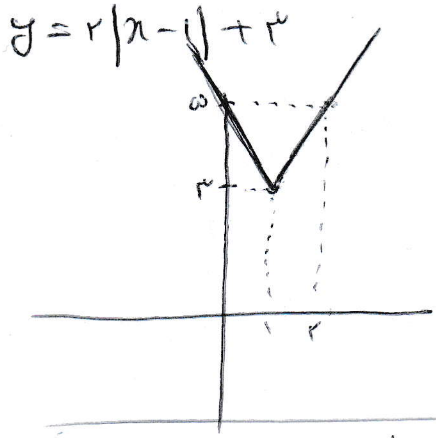
$$f(x)=\frac{|x|-[x]}{[x]^2-3[x]-10}$$

$$[x]^2-3[x]-10=0 \longrightarrow ([x]-5)([x]+2)=0 \begin{cases} [x]=5 \rightarrow x\in \boxed{[5,6)} \\ [x]=-2 \rightarrow x\in \boxed{[-2,-1)} \end{cases}$$

$$\longrightarrow D_f=\mathbb{R}-\left([-2,-1)\cup[5,6)\right)=\boxed{(-\infty,-2)\cup[-1,5)\cup[6,+\infty)}$$

10-

$y=2|x-1|+3$



این تابع در فاصله $[1,+\infty)$ یا $(-\infty,1]$

یک به یک می‌باشد. پس:

$$\boxed{a\geq 1}$$

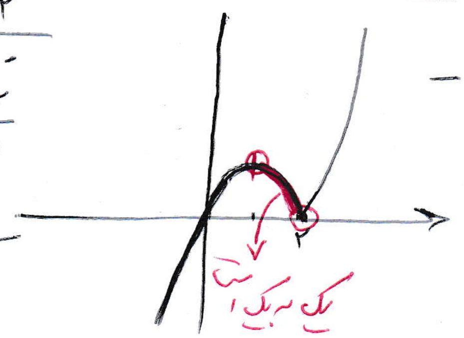
11-

$$y=x|x-2| = \begin{cases} x^2-2x & : x\geq 2 \\ -x^2+2x & : x<2 \end{cases}$$

x	0	1	2
y	0	-1	0

x	0	1	2
y	0	1	0

$D_f=(1,2)$



$R_f=(0,1)$

معکوس: $y=x|x-2| \xrightarrow{1<x<2} y=-x^2+2x$

$\xrightarrow{\text{معکوس}} x=-y^2+2y \longrightarrow y^2-2y+x=0 \longrightarrow (y-1)^2-1+x=0 \longrightarrow$

$(y-1)^2=-x+1 \longrightarrow y-1=\pm\sqrt{1-x} \longrightarrow \boxed{y=1\pm\sqrt{1-x} \quad ; \quad 0<x<1}$

$D_{f^{-1}}=R_f$

12-